BALASORE SCHOOL OF ENGINEERING

ENGINEERING MATHEMATICS :- III

THEORY-01

BRANCH :- ELECTRICAL AND E.T.C.

CHAPTER—I
COMPLEX NUMBER

1.a) Find the conjugate of $\frac{1}{1-i}$ (2013-s-1(iii))

Soln. Conjugate of $\frac{1}{1-i}$

Now $Z = \frac{1.(1+i)}{(1-i)(1+i)} = \frac{1+i}{1-i^2} = \frac{1+i}{1+1} = \frac{1+i}{2}$

$\Rightarrow Z = \frac{1}{2} + \frac{1}{2}i$

$\text{conjugate}\, Z = \frac{1}{2} - \frac{1}{2}i$

(b) Find x and y when (2014 (w) Q 1(ii))

Determinant short questions

$$\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

Now $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x+3y \\ 2x-y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$\Rightarrow x + 3y = 4 \longrightarrow (1)$

And $2x - y = 1 \longrightarrow (2)$

Eqn. $1 \times 2 \Rightarrow 2x + 6y = 8$

$2x - y = 1$

(-) (+) (-)

$7y = 7$

$\Rightarrow Y = 7/7 = 1$

$\therefore x + 3y = 4 \Rightarrow + 3.1 = 4 \Rightarrow x = 4 - 3 = 1$

$\therefore x = 1$ and $y = 1$ (Ans.)

2.a Find the square root of $-5 + 12 I$ / $-5 + 12\sqrt{-1}$ 2013 (w) Q 2(b)

Soln. Let $x, y \in R$,

So $x + Iy = \sqrt{-5+12i}$

$\Rightarrow (x + iy)^2 = -5 + 12i$

$\Rightarrow x^2 - y^2 + i2xy = -5 + 12i$

Equating real and imaginary parts we get

$X^2 - y^2 = -5$ and $2xy = 12$

We know that $(x^2 + y^2)^2 - (x^2 - y^2)^2 = 4x^2y^2$

$\Rightarrow (x^2 + y^2)^2 = (x^2 + y^2)^2 + 4x^2y^2$

$\Rightarrow (x^2 + y^2)^2 = (-5)^2 + (12)^2$

$\Rightarrow (x^2 + y^2)^2 = 25 + 144 = 169$

$\Rightarrow x^2 + y^2 = \sqrt{169} = 13$

$\therefore x^2 + y^2 = 13$

$X^2 - y^2 = -5$

$\Rightarrow 2x^2 = 8 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

And $x^2 + y^2 = 13 \Rightarrow 2^2 + y^2 = 13 \Rightarrow y^2 = 13 - 4 = 9$

$\Rightarrow y = \pm 3$

$\therefore \sqrt{-5 \mp 12i} = \pm (2+3i)$ (Ans)

(b) If l, ω, ω^2 are the three cube roots as unity then prove that

$(l-\omega+\omega^2)(l-\omega^2+\omega^4)(l-\omega^4+\omega^8)$ .... to 2n factors $= 2^{2n}$ 2016 (w) 2(d)

Soln. Now $(l-\omega+\omega^2)(l-\omega^2+\omega^4)(l-\omega^4+\omega^8)$.............. to 2n factors

$= (l+\omega^2-\omega)(l-\omega^2+\omega^3)(l-\omega.\omega^3+\omega^2.\omega^6)$...... to 2n factors

$= (l+\omega^2-\omega)(l-\omega^2+\omega.1)(l-\omega.1+\omega^2.1)$ .... to 2n factors

$= (-\omega-\omega)(l+\omega-\omega^2)(l + \omega^2-\omega)$ .... to 2n factors

$= (-2\omega)(-\omega^2-\omega^2)(-\omega-\omega)$................to 2n factors

$= (2^2\omega^3)(2^2\omega^3)(2^2\omega^3)$ .... to n factors

$= (2^2.l)(2^2.l) . (2^2.l)$................to n factors

$= 2^2 .2^2 . 2^2$ ....to n factors

$= 2^n$ (proved).

(c)If l, ω, ω^2 are cube roots of unity show $(l+ 5\omega^2 + \omega^4)(l+5\omega + \omega^2)(5 + \omega+\omega^2) = 64$ (**2017 W 2-C)**

Soln. $(l+5\omega^2+\omega^4)(l+5\omega+\omega^2)(5+\omega+\omega^2)$

$= (l+\omega^2+4\omega^2+\omega.\omega^3)(l+\omega+4\omega+\omega^2)(l+4+\omega+\omega^2)$

$= (l+\omega^2+4\omega^2+\omega.l)(l+\omega+4\omega+\omega^2)(l+\omega+\omega^2+4)$

$= (l+\omega+\omega^2+4\omega^2)(l+\omega+\omega^2+4\omega)(l+\omega+\omega^2+4)$

$= (0+4\omega^2)(0+4\omega)(0+4)$

$= (4\omega^2)(4\omega).(4)$

$= (4 \times 4 \times 4)(\omega^2.\omega)$

$=64.\ \omega^3 = 64 \times 1 = 64$ (R.H.S) (Proved).

03.a) Find the conjugate of $\frac{(2+3i)^2}{2-i}$ 2015 (w) Q-1.VI

Let $z = \frac{(2+3i)^2}{2-i}$

$= \frac{4+9i^2+12i}{2-i}$

$= \frac{4-9+12i}{2-i}$

$= \frac{12i-5}{2-i} = \frac{(12i-5)(2+i)}{(2-i)(2+i)}$

$= \frac{24i+12i^2-10-5i}{4+1}$

$= \frac{19i-22}{5}$

$= \frac{-22}{5} + \frac{19}{5}i$

So $\bar{Z} = \frac{-22}{5} - \frac{19}{5}i$

(c) If l,ω, ω^2 are three cube roots of unity, show that $(l, \omega + \omega^2)^5 + (l+\omega-\omega^2)^5 = 32$.

Ans: $(l-\omega+\omega^2)^5 + (l+\omega-\omega^2)^5$

$= (l + \omega^2 - \omega)^5 + (l + \omega - \omega^2)^5$ $(\because l+\omega^2 = -\omega, \quad l+\omega = -\omega^2)$

$= (-\omega - \omega)^5 + (-\omega^2 - \omega^2)^5$

$= (-2\omega)^5 + (-2\omega^2)^5 = (-2)^5 \{\omega^5 + \omega^{10}\}$

$= -32\ (\omega^2 + \omega)$ $\{\because \omega^5 = \omega^3 \times \omega^{2=}\ \omega^2 \quad \omega^{10} = (\omega^3)^3 \times \omega = \omega\omega^2 + \omega = -1\}$

$= -32\ (-l)$

= 32 R.H.S (Proved)

Long Question: **2017(W)-2C**

04. If $x + \frac{1}{x} = 2\cos\theta$, show that $x^n + \frac{1}{x^n} = 2\cos n\theta$

Ans: $x + \frac{1}{x} = 2\cos\theta$

$\Rightarrow x^2 + 1 = 2\cos\theta$

$\Rightarrow x^2 + \sin^2\theta + \cos^2\theta = 2x\cos\theta$

$\Rightarrow (x - \cos\theta)^2 = i^2 \sin^2\theta$

$\Rightarrow (x - \cos\theta) = \pm I \sin\theta$

$\Rightarrow x = \cos \pm I \sin\theta$

If $x = \cos\theta + I \sin\theta$, $x^n = \text{Cos } n\theta + I \sin n\theta$

$$\frac{1}{x^n} = \cos n\theta - I \sin n\theta$$

Adding $x^n + \frac{1}{x^n} = \cos n\theta + i \sin n\theta + \cos n\theta - i\sin n\theta = 2 \cos n\theta$ (pvd)

Substracting $x^n - \frac{1}{x^n} = \cos n\theta + I \sin n\theta - \cos n\theta + i \sin n\theta = 2i \sin n\theta$

<u>CHAPTER-2</u>

1. i) Define rank. Find the rank of the matrix.

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 4 & 8 & 2 \end{bmatrix}$$

2018 (1.a)

Ans : Rank of the matrix which is obtained by eliminating largest order of non-vanishing minor of the matrix.

Here $P(A) \leq \min(3, 3)$

$$Let\ |A| \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 4 & 8 & 2 \end{bmatrix}$$

$= \quad 1 \begin{bmatrix} 4 & 0 \\ 8 & 2 \end{bmatrix} -2 \begin{bmatrix} 2 & 0 \\ 4 & 2 \end{bmatrix}$

$= \quad (8-0) - 2(4-0)$

$= \quad 8-2.4$

$= \quad 8-8 = 0$

So, the sub matrix of A are $\begin{bmatrix} 4 & 0 \\ 8 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 8 & 2 \end{bmatrix}$ ---------- so on

Now $|A_1| = \begin{bmatrix} 4 & 0 \\ 8 & 2 \end{bmatrix} = 8 - 0 = 8 \neq 0$

So, Rank of A, $f(A) = 2$

ii) Find the value of K, So that the system of equations x-y = 3 and 2x – ky = 4 have a unique solution.

2015 (W) Q. 1 (i)

Ans : Here $x - y = 3$

$2x - ky = 4$

$$\therefore \begin{bmatrix} 1 & -1 \\ 2 & -k \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

Now $\begin{bmatrix} 1 & -1 \\ 2 & -k \end{bmatrix} = -k + 2 \neq 0$

$\Rightarrow \quad k \neq 2$

$\therefore$ The value of $k \neq 2$, the system of equation x – y =3 and 2x – ky = 4 have a unique solution.

iii) Define Rouchels theorem.

2013 (W) Q.1 (i)

Ans : The system of equation

$$a_{11}x_1 + a_{12}x_2 + \text{---------} + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \text{---------} + a_{2n}x_n = b_2$$

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$a_{m1}x_1 + a_{m2}x_2 + \text{---} + a_{mn}x_n = b_m$ is consistent if the co-efficient matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & --- & a_{1n} \\ a_{21} & a_{22} & --- & a_{2n} \\ --- & --- & --- & --- \\ a_{m1} & a_{m2} & --- & a_{mn} \end{bmatrix}$$ and the augmented

Matrix $$k = \begin{bmatrix} a_{11} & a_{12} & --- & a_{1n} \mid b_1 \\ a_{21} & a_{22} & --- & a_{2n} \mid b_2 \\ --- & --- & --- & --- \\ a_{m1} & a_{m2} & --- & a_{mn} \mid b_m \end{bmatrix}$$ are of same rank

Otherwise the system is inconsistent.

2. a) Test consistency and solve

5x + 3y + 7z = 4

3x + 26y + 2z = 9

7x + 2y + 10z = 5

2018 (W) 2.b

The system of equations can be written as

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

(By applying row – reduced Echelon form)

The given equation which can be represented in Augmented matrix form.

$$K = \begin{bmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3/5 & 7/5 & 4/5 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{bmatrix} \qquad \left(R_1 \to R_1/5 \right)$$

$$\sim\begin{bmatrix} 1 & 3/5 & 7/5 & 4/5 \\ 0 & \frac{-121}{5} & \frac{11}{5} & \frac{-33}{5} \\ 0 & \frac{11}{5} & \frac{-1}{5} & \frac{3}{5} \end{bmatrix} \qquad \begin{pmatrix} R_2 \to 3R_1 - R_2 \\ R_3 \to 7R_1 - R_3 \end{pmatrix}$$

$$\sim\begin{bmatrix} 1 & 3/5 & 7/5 & 4/5 \\ 0 & 1 & \frac{1}{-11} & \frac{3}{11} \\ 0 & \frac{11}{5} & \frac{-1}{5} & \frac{3}{5} \end{bmatrix} \qquad \left(R_2 \to R_2 \times \frac{5}{-121}\right)$$

$$\sim\begin{bmatrix} 1 & 3/5 & 7/5 & 4/5 \\ 0 & 1 & \frac{1}{-11} & \frac{3}{11} \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \left(R_3 \to R_2 \times \frac{11}{5} - R_3\right)$$

Here number of non-zero rows = 2

$\therefore \int(A) = \int(K) = 2 < 3$

Hence the system is consistent and infinitely many solutions.

Now $1.x + \frac{3}{5}.y + \frac{7}{5}.z = \frac{4}{5}$ ---------(1)

$1.y - \frac{1}{11}.z = \frac{3}{11}$ ---------(2)

Equation (2) $\Rightarrow$ $y = \frac{3}{11} + \frac{1}{11}z = \frac{3+z}{11}$

$\therefore$ Equation (1) $\Rightarrow$ $x + \frac{3}{5}\left(\frac{3+z}{11}\right) + \frac{7}{5}.z = \frac{4}{5}$

$$\Rightarrow x + \frac{9}{55} + \frac{3z}{55} + \frac{7}{5}.z = \frac{4}{5}$$

$$\Rightarrow x + \frac{3z + 77z}{55} = \frac{4}{5} - \frac{9}{55}$$

$$\Rightarrow x + \frac{80z}{55} = \frac{44-9}{55}$$

$$\Rightarrow x + \frac{16}{11}z = \frac{35}{55} = \frac{7}{11}$$

$$\Rightarrow x = \frac{7}{11} - \frac{16}{11}z$$

$$\Rightarrow x = \frac{7-16z}{11}$$

$\therefore$ $x = \frac{7-16z}{11}$ *and* $y = \frac{3+z}{11}$

Z is a parameter

3. Investigate for what value of λ and μ, the simultaneous equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

have (i) no solution

(ii) a unique solution **[2015 (W) Q. (4)]**

(iii) An infinite number of solutions. **[2014 (W) Q. 7(c)]**

Solution Here

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

$$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

Here $K = \begin{bmatrix} 1 & 1 & 1 & | \; 6 \\ 1 & 2 & 3 & | \; 10 \\ 1 & 2 & \lambda & | \; \mu \end{bmatrix}$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda - 1 & \mu - 6 \end{array}\right] \qquad \begin{pmatrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{pmatrix}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda - 3 & \mu - 10 \end{array}\right] \qquad \begin{pmatrix} R_1 \to R_1 - R_2 \\ R_3 \to R_3 - R_2 \end{pmatrix}$$

The system has no solution.

Case –I If $\lambda \neq 3$ and $\mu \neq 10$ then the rank of co-efficient matrix = 2 and the rank of the augmented matrix = 3.

So $f(A) \neq f(K)$

The system has no solution.

Case –II If $\lambda \neq 3$ and μ may have any value, then the rank $f(A) = 3$ and $f(K) = 3$

So $f(A) = f(K)$ = no.of unknowns. (3)

So the system has unique solution.

Case – III If $\lambda = 3$ and $\mu = 10$ then

$f(A) = 2$ and $f(K) = 2$

$\therefore f(A) = f(K) = 2 <$ no. of unknowns (3)

So the system has an infinite number of solutions.

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CHAPTER-3

1. (i) If the roots of the d.e. are $\frac{-1+\sqrt{3}i}{2}$ and $\frac{-1-\sqrt{3}i}{2}$ then find the C.F.

Ans : If, $m_1 = \frac{-1+\sqrt{3}i}{2}, \quad m_2 = \frac{-1-\sqrt{3}i}{2}$ **[2013 (W), Q. 1 (b)]**

Roots are imaginary and different.

So, C.F. $(y_c) = \left(c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x \right) e^{\frac{-1}{2}x}$

ii) If $(D^2 - 1)y = 0$

Ans ; **[2018 (W), Q. 1 (b)]**

$D^2 - 1 = 0$

⇨ $D = \pm 1$

⇨ $y = c_1 e^x + c_2 e^{-x}$

iii) Solve : $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$ 2016(S)1.V

Ans : $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$

$\Rightarrow (D^2y + 6Dy + 9y) = 0$

$\Rightarrow y(D^2 + 6D + 9) = 0$

A.E. is $D^2 + 6D + 9 = 0$

$\Rightarrow (D+3)^2 = 0$

$\Rightarrow D = -3, -3$

Roots are real and equal.

So, C.F. $(y_c) = (C_1 + C_2x)\, e^{-3x}$

iv) Solve $xp + yq = z$ [2018, 1(e)]

Ans: $xp + yq = z$

A.E. is $\frac{dx}{p} = \frac{dy}{q} = \frac{dz}{R}$

$\Rightarrow \quad \frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$

Considering 1st two ratios $\frac{dx}{x} = \frac{dy}{y}$

Now integrating both sides.

$\int \frac{dx}{x} = \int \frac{dy}{y}$

$\Rightarrow \ln x = \ln y + \ln c_1$

$\Rightarrow \ln x = \ln yc_1$

$\Rightarrow x = yc_1$

$\Rightarrow c_1 = x/y$

Similarly considering last 2 ratios,

$$\frac{dy}{y} = \frac{dz}{z}, \text{ we get}$$

$$c_2 = {y}/{z}$$

So, Solution is F (c_1, c_2) = 0

$$\Rightarrow F\left(\frac{x}{y}, \ \frac{y}{x}\right) = 0$$

$$\frac{x}{y} = f\left(\frac{y}{z}\right).$$

v) Form a P.D.E. by eliminating the arbitrary function [2018, 2(d)]

$Z = f(x^2 - y^2)$

Ans : $Z = f(x^2 - y^2)$

Diff. partially we get $\frac{\partial z}{\partial x} = f^1(x^2 - y^2) \times 2x$ ---------(1)

Diff. partially we get $\frac{\partial z}{\partial y} = f^1(x^2 - y^2)(-2y)$ ---------(2)

Dividing (1) and (2) $\frac{\partial z}{\partial x} \Big/ \frac{\partial z}{\partial y} = \frac{f'(x^2-y^2)2x}{f'(x^2-y^2)(-2y)} - x/y$

$$\Rightarrow y\frac{\partial z}{\partial x} = -x\frac{\partial z}{\partial y}$$

$$\Rightarrow x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x} = 0$$

$$\Rightarrow xp + yq = 0$$

This is the required P.D.E.

2. (i) Solve : $\frac{d^2y}{dx^2} + 4y = x^2 + \cos 2x$

2014(W)3.C

Ans : $$\frac{d^2y}{dx^2} + 4y = x^2 + \cos 2x$$

$\Rightarrow (D^2y + 4y) = x^2 + \cos 2x$

$\Rightarrow (D^2+4)y = x^2 + \cos 2x$

C.F. A.E. $(D^2+4) = 0$

$\Rightarrow D = \sqrt{-4} = \pm 2i$

$y_c = (c_1\cos 2x + c_2 \sin 2x)\, e^{0.x}$

$= (c_1\cos 2x + c_2 \sin 2x)$

Now, to find P.I.

$(D^2+4)\, y = x^2 + \cos 2x.$

$$\Rightarrow y_p = \frac{x^2 + \cos 2x}{D^2+4}$$

$$= \frac{x^2}{D^2+4} + \frac{\cos 2x}{D^2+4}$$

$$= P.I._1 + P.I_2$$

$$P.I_1 = \frac{x^2}{D^2+4} = \frac{x^2}{4\left(\frac{D^2+4}{4}\right)}$$

$$= \frac{x^2}{4\left(1+\frac{D^2}{4}\right)} = \frac{1}{4}\left(1+\frac{D^2}{4}\right)^{-1} x^2$$

$$= \frac{1}{4}\left[1-\frac{D^2}{4}+\frac{D^4}{16}-----\right] \quad x^2$$

$$= \frac{1}{4}\left[x^2-\frac{D^2}{4}x^2+\frac{D^4}{16}x^2\right]$$

$$= \frac{1}{4}\left[x^2-\frac{2}{4}+0\right]$$

$$= \frac{1}{4}\left[x^2-\frac{1}{2}\right]$$

$$P.\mathrm{I}_2 = \frac{\cos 2x}{\mathrm{D}^2+4} = \frac{\cos 2x}{-2^2+4} = \frac{\cos 2x}{-4+4}$$

So, the formula fails.

$$So,\ P.\mathrm{I}_2 = \mathrm{x}\,\frac{\cos 2x}{\mathrm{F}^1(R^2)} = \mathrm{x}\,.\frac{\cos 2x}{2D}$$

F (D) = D^2+4
F^1 (D) = 2D
F^1 (R) = 2D

$$= \frac{x}{2}\frac{1}{D}.\cos 2x$$

$$= \frac{x}{2}\int \cos 2x dx.$$

$$= \frac{x}{2}\ \frac{\sin 2x}{2}.$$

$$= \frac{x}{4}\ \sin 2x$$

$$y_p = P.I_1 + P.I._2$$

$$= \frac{1}{4}\left(x^2-\frac{1}{2}\right)+\frac{x}{4}\sin^2 x$$

$$y = y_c + y_p = (c_1\cos 2x + c_2 \sin 2x) + \frac{1}{4}(x^2-\frac{1}{2}) + \frac{x}{4}\sin 2x$$

ii) Solve : $\frac{d^{3y}}{dx^3} + 4\frac{dy}{dx} = \sin 2x$

Ans : $\frac{d^{3y}}{dx^3} + 4\frac{dy}{dx} = \sin 2x$ **[2014 (W) Q. 2(b)]**

$$\Rightarrow (D^3 y + 4Dy) = \sin 2x$$

$$\Rightarrow y(D^3+4D) = \sin 2x.$$

$$A.E.\ is \quad D^3+4D=0$$

$$\mathrm{D}\ (D^2+4)=0$$

$$\Rightarrow D=0 \qquad \mathrm{D}^2+4=0$$

$D^2 + 4 = 0 \quad \Rightarrow D\sqrt{-4}$
$= \pm 2i$
$y_c = (c_1 \cos 2x + c_2 \sin 2x)e^{0.x}$
$= (c_1 \cos 2x + c_2 \sin 2x)$
$P.I. \quad (D^3 + 4D)y = \sin 2x$

$$\Rightarrow y = \frac{\sin 2x}{D^3 + 4D}$$

$$y_p = \frac{\sin 2x}{D^2.D + 4D}$$

$$= \frac{\sin 2x}{-4D + 4D}$$

So, the formula fails.

$$y_p = x.\frac{\sin 2x}{F^1.(2^2)}$$

$$= \frac{x}{-8}\sin x.$$

$y = y_c + y_p$

$$= (c_1 \cos 2x + c_2 \sin 2x) - \frac{x}{8}\sin 2x$$

3 (a) Solve x $(y^2 - z^2)$ p +y $(z^2 - x^2)$ = z (x^2-y^2) **[2018 (W) Q. 4]**

Ans : x $(y^2 - z^2)$ p +y $(z^2 - x^2)$ = z (x^2-y^2)

A.E. is $\frac{dx}{p} = \frac{dy}{Q} = \frac{dz}{R}$

$$\Rightarrow \frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}$$

Choosing the multiplier $P^1 = \frac{1}{x}$

$Q^1 = \frac{1}{y}$

$R^1 = \frac{1}{z}$

So that $PP^1 + QQ^1 + RR^1$

$$= x(x^2 - z^2) \times \frac{1}{x} + y(z^2 - x^2) \times \frac{1}{y} + z(x^2 - y^2) \times \frac{1}{z}$$

$= y^2 - z^2 + z^2 - x^2 + x^2 - y^2 = 0$

So each ratio $= \frac{P^1 dx + Q^1 dy + R^1 dz}{PP^1 + QQ^2 + RR^1}$

$$\Rightarrow \frac{dx}{x(y^2 - z^2)} = \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{0}$$

$$\Rightarrow \frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$$

Now, integrating both sides, we get

$$\int \frac{1}{x}dx + \int \frac{1}{y}dy + \int \frac{1}{z}dz = \log c_1$$

$\Rightarrow log\ x + log\ y + log\ z = log\ c_1$

$\Rightarrow ln\ \ xyz = ln\ c_1$

$\Rightarrow xyz = c_1$ ------------------ (1)

Again, choosing the multiplier as $P'' = x$
$Q'' = y$
$R'' = z$

So $PP'' + QQ'' + RR''$
$= x^2 (y^2 - z^2) + y^2(z^2 - x^2) + z^2 (x^2 - y^2) = 0$

So, each ratio = $\dfrac{P''dx + Q''dy + R''dz}{PP'' + QQ'' + RR''}$

$= \dfrac{xdx + ydy + zdz}{0}$

$\Rightarrow xdx + ydy + zdz = 0.$

Integrating both sides, we get

$$\int xdx + \int ydy + \int zdz = c_2$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = c_2$$

$$\Rightarrow \left(x^2 + y^2 + z^2\right) = 2c_2 = c_3$$

So, the solution of $F(c_1, c_2) = 0$

$$\Rightarrow F\left(x\ y\ z,\ \ x^2 + y^2 + z^2\right) = 0$$

$$\Rightarrow \ \ x\ y\ z = f\left(\ x^2 + y^2 + z^2\right)$$

b) Solve : $\dfrac{d^2y}{dx^2} + 4y = e^{-x}\sin x + x$

$\Rightarrow \left(D^2y + 4y\right) = e^{-x}\sin x + x$

A.E. $D^2+4 = 0 \quad \Rightarrow D^2 = -4 \ \ \Rightarrow D = \pm 2i$

$y_c = (c_1\cos 2x + c_2 \sin 2x)\ e^{0.x}$

$= (c_1\cos 2x + c_2 \sin 2x)$

P.I. $(D^2+4)y = e^{-x} \sin x + x$

$$\Rightarrow \ \ y = \frac{e^{-x}\sin x}{D^2 + 4} + \frac{x}{D^2 + 4}$$

$$y_p = \frac{e^{-x}\sin x}{(D-1)^2 + 4} + \frac{x}{4\left(\dfrac{D^2+4}{4}\right)}$$

$$= e^{-x}\frac{\sin x}{D^2 + 1 - 2D + 4} + \frac{x}{4\left(\dfrac{1+D^2}{4}\right)}$$

$$= e^{-x}\frac{\sin x}{D^2-2D+5}+\frac{1}{4}\left[1+\frac{D^2}{4}\right]^{-1}x$$

$$=\frac{e^{-x}\sin x}{5\left(\frac{D^2-2D+5}{5}\right)}+\frac{1}{4}\left[1-\frac{D^2}{4}+\frac{D^4}{16}\right]x$$

$$\Rightarrow \frac{e^{-x}}{5}\left[1+\frac{D^2-2D}{5}\right]^{-1}\sin x+\frac{1}{4}\left(x-\frac{D^2}{4}.x\right)$$

$$=\frac{e^{-x}}{5}\left[1-\frac{(D^2-2D)}{5}+\left(\frac{D^2-2D}{5}\right)^2.....\right]\sin x+\frac{1}{4}(x-0)$$

$$=\frac{e^{-x}}{5}\left(\sin x-\left(\frac{D^2-2D}{5}\right)\sin x\right)+\frac{1}{4}x$$

$$=\frac{e^{-x}}{5}\left(\sin x+\frac{\sin x+2\cos x}{5}\right)+\frac{1}{4}x$$

$$=\frac{e^{-x}}{5}\left(\frac{6\sin x+2\cos x}{5}\right)+\frac{1}{4}x$$

$$=\frac{e^{-x}}{25}(6\sin x+2\cos x)+\frac{1}{4}x$$

Solution is $y = y_c + y_p$

$$=(c_1\cos 2x+c_2\sin 2x)+\frac{e^{-x}}{25}(6\sin x+2\cos x)+\frac{1}{4}x$$

3 (c) Solve : $(D^2+4)y = e^x \sin^2 x$

Ans : C.F A.E. is $D^2+4 = 0$ $\Rightarrow D^2 = -4$ **[2014 (W) Q. 1(c)]**

$\Rightarrow D=\pm 2i$

C.F = $y_c = c_1\cos 2x + c_2 \sin 2x$

P.I. $(D^2+4)y = e^{-x} \sin^2 x$

$$\Rightarrow y=\frac{e^x\sin^2 x}{D^2+4}$$

$$=\frac{e^x(1-\cos 2x)}{2(D^2+4)}$$

$$=\frac{1}{2}\left[\frac{e^x}{D^2+4}-\frac{e^x\cos 2x}{D^2+4}\right]$$

$$y_p=\frac{1}{2}\left[\frac{e^x}{(1)^2+4}-\frac{e^x\cos 2x}{(D+1)^2+4}\right]$$

$$=\frac{1}{2}\left[\frac{e^x}{5}-e^x\frac{\cos 2x}{D^2+2D+1+4}\right]$$

$$=\frac{1}{2}\left[\frac{e^x}{5}-e^x\frac{\cos 2x}{D^2+2D+5}\right]$$

$$=\frac{1}{2}\left[\frac{e^x}{5}-e^x\frac{\cos 2x}{-4+2D+5}\right]$$

$$=\frac{1}{2}\left[\frac{e^x}{5}-e^x\frac{\cos 2x}{2D+1}\right]$$

$$=\frac{1}{2}\left[\frac{e^x}{5}-e^x\frac{\cos 2x(2D-1)}{4D^2-1}\right]$$

$$=\frac{1}{2}\left[\frac{e^x}{5}-\frac{e^x}{4.(-4)-1}\cos 2x(2D-1)\right]$$

$$=\frac{1}{2}\left[\frac{e^x}{5}+\frac{e^x}{17}(2D\cos 2x-\cos 2x)\right]$$

$$=\frac{1}{2}\left[\frac{e^x}{5}+\frac{e^x}{17}(-4\sin 2x-\cos 2x)\right]$$

Hence G.S. is

y = yc + yp

$$=c_1\cos 2x+c_2\sin 2x+\frac{1}{2}\left[+\frac{e^x}{5}-\frac{e^x}{17}(4\sin x+\cos 2x)+\frac{1}{4}x\right]$$

3. (d) Solve : $\frac{d^2y}{dx^2}+16y=x\sin 3x$

Ans : C.F. A.E is $(x^2 + I6) = 0$ **[2014 (W) Q. 3 (b)]**

$\Rightarrow D^2 = -16$

$\Rightarrow D = \pm 4i$

C.F. = y_c = $c_1\cos 4x + c_2 \sin 4x$

P.I.y $(D^2 + 16)$ = x sin 3x

$$\Rightarrow y=\frac{x\sin 3x}{D^2+16}$$

$$y_p=\frac{1}{D^2+16}x\,(I.P\ of\ \ e^{3ix})$$

$$=I.P.\ of\ \frac{1}{D^2+16}e^{3ix}.x$$

$$=I.P.\ of\left[e^{3ix}\frac{1}{(D+3i)^2+16}x\right]$$

$$=I.P.\ of\left[e^{3ix}\frac{1}{D^2+6iD-9+16}x\right]$$

$$=I.P.\ of\left[e^{3ix}\frac{1}{D^2+6iD+7}x\right]$$

$$=I.P.\ of\left[\frac{e^{3ix}}{7}\frac{1}{\left(1+\frac{D^2+6iD}{7}\right)}x\right]$$

$$=I.P.\ of\left[\frac{e^{3ix}}{7}\left[1+\frac{D^2+6iD}{7}\right]^{-1}x\right]$$

$$= I.P.\ of \left[\frac{e^{3ix}}{7}\left(1+\frac{D^2+6iD}{7}+....\right)x\right]$$

$$= I.P.\ of \left[\frac{e^{3ix}}{7}\left(x+\frac{(D^2+6iD)x}{7}....\right)\right]$$

$$= I.P.\ of \left[\frac{e^{3ix}}{7}\left(x+\frac{6i}{7}\right)\right]$$

$$= I.P.\ of \left[\frac{\cos 3x + i\sin 3x}{7}\left(x+\frac{6i}{7}\right)\right]$$

$$= I.P.\ of \left[\frac{\cos 3x + i\sin 3x}{7}\times\left(\frac{7x+6i}{7}\right)\right]$$

$$= I.P.\ of \frac{1}{49}\left[(\cos 3x + i\sin 3x)(7x+6i)\right]$$

$$= I.P.\ of \frac{1}{49}\left[7x\cos 3x + 7ix\sin 3x + 6i\cos 3x + 6i^2\sin 3x\right]$$

$$= I.P.\ of \frac{1}{49}\left[7x\cos 3x - 6\sin 3x + i(7x\sin 3x + 6\cos 3x)\right]$$

$$= \frac{1}{49}(7x\cos 3x - 6\sin 3x)$$

3. (e) Solve : $xp - yq = y^2 - x^2$ 2016(W) 5.C

Ans : The subsidiary equations are

$$\frac{dx}{x} = \frac{dy}{-y} = \frac{dz}{y^2-x^2}$$

Considering 1st two ratios, we get

$$\frac{dx}{x} = \frac{dy}{-y}$$

Which on integration

$$\int \frac{dx}{x} = -\int \frac{dy}{y}$$

$$\Rightarrow \log x = -\log y + \log c$$

$$\Rightarrow \log x + \log y = \log c$$

$$\Rightarrow xy = c \qquad ------(1)$$

Using multipliers x, y & 1 e have

Each fraction = $\dfrac{xdx + ydy + 1dz}{x^2 - y^2 + y^2 - x^2}$

$$\Rightarrow \frac{xdx + ydy + 1dz}{0} = \frac{dx}{x}$$

$$\Rightarrow xdx + ydy + dz = 0$$

Integrating both sides, we get

$$\int xdx + \int ydy + \int dz = c_2$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} + z = c_2$$

$$\Rightarrow (x^2 + y^2 + 2z) = c_2 \qquad ---------(2)$$

From (I) and (2) $f(xy, x^2 + y^2 + 2z) = 0$

4. (a) Find the complementary function of $(D^2-2D+2)y = \sin 3x$ 2014 Q3(a)

Solution : $(D^2-2D+2)y = 0$

It's A.E is

$\Rightarrow D^2-2D+2 = 0$

$$D = \frac{-(-2) \pm \sqrt{(-2)^2 - 4.1.2}}{2 \times 1}$$

$$= \frac{2 \pm \sqrt{4-8}}{2}$$

$$= \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$$= 1 \pm i$$

$$y_c = (c_1 \cos x + c_2 \sin x)\ e^x$$

b) From the partial differential equation by eliminating arbitrary functions from [2016 (S),2.B]

$$Z = f\left(\frac{y}{x}\right)$$

Ans : $Z = f\left(\frac{y}{x}\right)$ ---------(1)

Differentiating the equation (1) partially w.r.t 'x' and also 'y'

$$\frac{\partial z}{\partial x} = f'\left(\frac{y}{x}\right)\left(\frac{-y}{x^2}\right)$$

$$\Rightarrow P = \frac{-y}{x^2} f'\left(\frac{y}{x}\right) \text{---------(2)}$$

$$\frac{\partial z}{\partial y} = f'\left(\frac{y}{x}\right).\frac{1}{x}$$

$$\Rightarrow q = \frac{1}{x} f'\left(\frac{y}{x}\right) \text{---------(3)}$$

Divide the equation (2) by the equation (3) we have

$$\frac{p}{q} = \frac{\frac{-y}{x^2} f'\left(\frac{y}{x}\right)}{\frac{1}{x} f'\left(\frac{y}{x}\right)}$$

$$\Rightarrow \frac{p}{q} = \frac{-y}{x}$$

$\Rightarrow px = - qy$

$\Rightarrow px + qy = 0$ is the required partial differential equation of 1st order.

4. (c) Find particular integral of $(D^2 + D + 1)\ y = \cos 2x$

Solution : $(D^2 + D + 1)\ y = \cos 2x$

To find P.I. **[2015 (w) Q. 2 (b)]**

$$\frac{1}{D^2 + D + 1} \cos 2x$$

$$= \frac{1}{-4 + D + 1} \cos 2x$$

$$= \frac{1}{D-3}\cos 2x$$

A = 2
D2 = -(2) = -4

$$= \frac{(D+3)}{(D-3)(D+3)}\cos 2x$$

$$= \frac{(D+3)}{D^2-9}\cos 2x$$

$$= \frac{(D+3)}{-4-9}\cos 2x$$

$$= -\frac{1}{13}(D\cos 2x + 3\cos 2x)$$

$$= -\frac{1}{13}(-2\sin 2x + 3\cos 2x)$$

$$= \frac{1}{13}(2\sin 2x - 3\cos 2x)$$

5. (a) Solve : [2016(W),4.B]

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$$

Let $\frac{d}{dx} = D, \frac{d^2}{dx^2} = D^2$

$\Rightarrow \quad D^3y - 2D^2y + 4Dy - 8y = 0$

$\Rightarrow \quad (D^3 - 2D^2 + 4D - 8)y = 0$

It's A.E. is

$\Rightarrow \quad D^3 - 2D^2 + 4D - 8 = 0$

$\Rightarrow \quad D^2(D-2) + 4(D-2) = 0$

$\Rightarrow \quad (D-2)(D^2+4) = 0$

$\Rightarrow \quad (D-2) = 0 \quad$ or $\quad D^2 + 4 = 0$

$\Rightarrow \quad D = 2 \qquad \Rightarrow \quad D^2 = -4$

$\Rightarrow \quad D = \sqrt{-4}$

$\Rightarrow \quad D = \pm 2i$

$y = c_1e^{2x} + (c_2\cos 2x + c_3\sin 2x)e^{0.x}$

$\Rightarrow y = c_1e^{2x} + c_2\cos 2x + c_3\sin 2x$

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CHAPTER-4

1 (a) $L\ (\sin^2 2t)$ **[2013 (w) Q. -1(iii)]**

Ans : $= L\left(\frac{1-\cos 4t}{2}\right)$

$= \frac{1}{2}\ [L\,(1-\cos 4t)]$

$$= \frac{1}{2}\left[L(1) - L(\cos 4t)\right]$$

$$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+(4)^2}\right]$$

$$= \frac{1}{2}\left[\frac{s^2+16-s^2}{s(s^2+16)}\right]$$

$$= \frac{8}{s(s^2+16)}$$

(b) Find $L^{-1}\left(\frac{s}{a^2s^2+b^2}\right)$ **[2017 (w) Q. – 1 (viii)]**

$$= L^{-1}\left(\frac{s}{a^2\left(\frac{a^2s^2+b^2}{a^2}\right)}\right)$$

$$= \frac{1}{a^2}L^{-1}\left(\frac{s}{s^2+b^2/a^2}\right)$$

$$= \frac{1}{a^2}L^{-1}\left(\frac{s}{s^2+(b/a)^2}\right)$$

$$= \frac{1}{a^2}\cos\frac{b}{a}\,t$$

(c) $L\left(e^{2t}.t^5\right)$ **[2018(w) Q. – 1 (c)]**

$$= \frac{5!}{(s-2)^{5+1}} = \frac{120}{(s-2)^6}$$

(d) $L\left(\sin t - \cos t\right)^2$ **[2012 (w) Q. – 1 (vii)]**

$$= L\left(\sin^2 t + \cos^2 t - 2\sin t.\cos t\right)$$

$$= L\left(1 - \sin 2t\right)$$

$$= L(1) - L(\sin 2t)$$

$$= \frac{1}{s} - \frac{2}{s^2+4}$$

$$= \frac{s^2+4-2s}{s\left(s^2+4\right)}$$

(e) $L\left(\cos(at+b)\right)$ **[2014(w) Q. – 1 (iii)]**

$$= L\left(\cos at.\cos b - \sin at.\sin b\right)$$

$$= L\left(\cos at.\cos b\right) - L\left(\sin at.\sin b\right)$$

$$= \cos b L\left(\cos at\right) - \sin b L\left(\sin at\right)$$

$$= \cos b \times \left(\frac{s}{s^2+a^2}\right) - \sin b\left(\frac{a}{s^2+a^2}\right)$$

$$= \frac{1}{s^2+a^2}\left[s\cos b - a\sin b\right]$$

(f) Find $L^{-1}\left(\frac{s^2-3s+4}{s^3}\right)$ **[2013(w) Q. – 1 (viii)]**

$$= L^{-1}\left(\frac{s^2}{s^3}-\frac{3s}{s^3}+\frac{4}{s^3}\right)$$

$$= L^{-1}\left(\frac{1}{s}\right)-3L\left(\frac{1}{s^2}\right)+\frac{4}{2}L^{-1}\left(\frac{2L}{s^3}\right)$$

$$=1-3t+\frac{4}{2}t^2=1-3t+2t^2$$

g) Find $L^{-1}\left(\frac{1}{(s-1)(s+3)}\right)$ **[2013 (w) Q. – 1 (ix)]**

$$= L^{-1}\left(\frac{1}{s-1}.\frac{1}{s+3}\right)$$

$$= L^{-1}\left(\frac{1}{s-1}\right)=e^t=f(t)$$

$$= L^{-1}\left(\frac{1}{s+3}\right)=e^{-3t}=g(t)$$

$$= L^{-1}\left(f(t).g(t)\right)=\int_o^t f(t-u)g(u)du$$

$$=\int_o^t e^{t-u}.e^{-3u}du$$

$$\int_o^t e^{t-u-3u}du=\int_o^t e^{t-4u}du$$

$$=e^t\int_o^t e^{-4u}du$$

$$=e^t\left[\frac{e^{-4u}}{-4}\right]_o^t$$

$$=e^t\left[\frac{e^{-4t}}{-4}+\frac{e^o}{4}\right] \qquad =e^t\left[\frac{1}{4}-\frac{e^{-4t}}{4}\right]$$

$$=\frac{e^t}{4}\left(1-e^{-4t}\right)$$

2 (i) $L=\left(\frac{1-e^t}{t}\right)$

$$= \int_s^\infty L(1-e^t)ds$$

$$= \int_s^\infty \left[L(1)-L(e^t)\right]ds$$

$$= \int_s^\infty \left(\frac{1}{s}-\frac{1}{s-1}\right)ds$$

$$= \int_s^\infty \frac{1}{s}\, ds - \int_s^\infty \frac{1}{s-1}\, ds$$

$$= \left[\ln \frac{s}{s-1}\right]_s^\infty$$

$$= \ln\frac{\infty}{\infty} - \ln\frac{s}{s-1}$$

$$= -\ln\frac{s}{s-1}$$

(ii) $L^{-1} = \dfrac{4s+5}{(s-2)(s+1)^2}$ [2015(W),7.C]

By partial traction :

$$\frac{4s+5}{(s-2)(s+1)^2} = \frac{A}{s-2} + \frac{B}{s+1} + \frac{C}{(s+1)^2} - - - - - - - - (1)$$

$$\Rightarrow \frac{4s+5}{(s-2)(s+1)^2} = \frac{A(s+1)^2 + B(s-2)(s+1) + C(s-2)}{(s-2)(s+1)^2}$$

$$\Rightarrow 4s+5 = A(s+1)^2 + B(s-2)(s+1) + C(s-2)$$

By putting s+1 = 0

$\Rightarrow$ s = -1

We get

4x(-1) +5 = 0 + 0+C (-1-2)

$\Rightarrow$ 1= -3c

$\Rightarrow$ C = -1/3

s-2= 0 $\Rightarrow$ s=2

4x2+5 = A $(2+1)^2$+0+0

$\Rightarrow$ 13=9A

$\Rightarrow$ A =13/9

The equation (1), becomes

$$4s + 5 = A(s^2 + 2s + 1) + B(s^2 - s - 2) + C(s - 2)$$

Equating co-eff. of s^2 0 = A+B

$\Rightarrow$B = -A = -13/9

By putting the vol. of A, B, & C in equation (1)

$$\frac{4s+5}{(s-2)(s+1)^2} = \frac{-13/9}{s-2} + \frac{13/9}{s+1} - \frac{\frac{1}{3}}{(s+1)^2}$$

$$L^{-1} = \left(\frac{4s+5}{(s-2)(s+1)^2}\right) = \frac{-13}{9}L^{-1}\left(\frac{1}{s-2}\right) + \frac{13}{9}L^{-1}\left(\frac{1}{s+1}\right) - \frac{1}{3}L^{-1}\left(\frac{1}{(s+1)^2}\right)$$

$$= \frac{13}{9}e^{2t} + \frac{13}{9}e^{-t} - \frac{1}{3}e^{-t}.t$$

(iii) Solve the following equation by Laplace transform method

$y'' + 4y' + 3y = e^{-t}$, $y(o) = y'(o) = 1$ [2016(S),3.C]

Ans : $y'' + 4y' + 3y = e^{-t}$

Taking Laplace transt^n of the given equation, we get

$$L(y'' + 4y' + 3y) = L(e^{-t})$$

$$\Rightarrow L(y'') + 4L(y') + 3L(y) = L(e^{-t})$$

$$\Rightarrow \left(s^2 Lf(y) - sf'(o)\right) - f'(o) - 4(sLf(y) - f(o) + 3L(f(y)) = \frac{1}{s+1}$$

$$\Rightarrow \left(s^2 f(s) - s.1 - 1\right) + 4\left(sf(s) - 1\right) + 3f(s) = \frac{1}{s+1}$$

$$\begin{pmatrix} \because y(o) = f(o) = 1 \\ y'(o) = f'(o) = 1 \end{pmatrix}$$

$$\Rightarrow \left(s^2 f(s) - s - 1 + 4sf(s) - 4 + 3f(s)\right) = \frac{1}{s+1}$$

$$\Rightarrow f(s)(s^2 + 4s + 3) - s - 5 = \frac{1}{s+1}$$

$$\Rightarrow f(s)(s^2 + 4s + 3) = \frac{1}{s+1} + s + 5$$

$$\Rightarrow f(s) = \frac{1}{(s+1)(s^2+4s+3)} + \frac{s+5}{(s^2+4s+3)}$$

$$\Rightarrow Lf(y) = \frac{1}{(s+1)(s^2+4s+3)} + \frac{s+5}{s^2+4s+3}$$

$$\Rightarrow f(y) = L^{-1}\left(\frac{1}{(s+1)(s+3)(s+1)} + \frac{s+5}{(s+3)(s+1)}\right)$$

$$= L^{-1}\left(\frac{1+(s+5)(s+1)}{(s+1)^2(s+3)}\right)$$

$$= L^{-1}\left(\frac{1+s^2+6s+5}{(s+1)^2(s+3)}\right)$$

$$= L^{-1}\left(\frac{s^2+6s+6}{(s+1)^2(s+3)}\right)$$

$$\frac{s^2+6s+6}{(s+1)^2(s+3)} = \frac{A}{s+3} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$$\Rightarrow \frac{s^2+6s+6}{(s+1)^2(s+3)} = \frac{A(s+1)^2 + B(s+1)(s+3) + C(s+3)}{(s+3)(s+1)^2}$$

$$\Rightarrow s^2 + 6s + 6 = A(s+1)^2 + B(s+1)(s+3) + C(s+3)$$

$s+1 = 0 \quad \Rightarrow s = -1$	$s+3 = 0 \quad \Rightarrow s = -3$
putting the value of S in eqn. (1)	Putting the val. Of S = -3 in eqn. (1)
$1+6(-1)+6 = 0 + 0 + ((-1) + 3)$	$9+6(-3)+6 = A(-3+1)^2 + 0 + 0$
$\Rightarrow \; 1 = 2c$	$-3 = 4A$
$\Rightarrow \; C = 1/2$	$A = -3/4$

Now, $s^2 + 6s + 6 = A(s^2+2s+1) + B(s^2+4s+3) + C(s+3)$

Equating the co-eff. of s^2

$1 = A+B \quad \Rightarrow B = 1-A = 1+3/4 = 7/4.$

$$So, \quad L^{-1}\left(\frac{s^2+6s+6}{(s+1)^2(s+3)}\right) = L^{-1}\left(\frac{A}{s+3} + \frac{B}{s+1} + \frac{C}{(s+1)^2}\right)$$

$$\Rightarrow \quad = L^{-1}\left(\frac{-3/4}{s+3}\right) + L^{-1}\left(\frac{7/4}{s+1}\right) + L^{-1}\left(\frac{1/2}{(s+1)^2}\right)$$

$$= \frac{-3}{4}e^{-3t} + \frac{7}{4}e^{t} + \frac{1}{2}e^{-t}.t$$

$$So, \quad (y) = \frac{-3}{4}e^{-3t} + \frac{7}{4}e^{-t} + \frac{1}{2}e^{-t}.t$$

3. (a) Find $L\left\{\frac{1}{\sqrt{t}}\right\}$

2014(W),1.IV

$$= \frac{\Gamma\left(-\frac{1}{2}+1\right)}{s^{-\frac{1}{2}+1}}$$

$$= \frac{\Gamma\left(-\frac{1}{2}\right)}{s^{\frac{1}{2}}} = \frac{\sqrt{\pi}}{s^{y2}}$$

(b) Find the Laplace tyransformof $(e^{at} - \cos bt)/t$

$$= L(e^{at} - \cos bt)/t$$

(2017(S),4.B)

$$= \int_s^\infty L(e^{at} - \cos bt)ds$$

$$= \int_s^\infty \left(\frac{1}{s-a} - \frac{s}{s^2+b^2}\right)ds$$

$$= \int_s^\infty \frac{1}{s-a}ds - \int_s^\infty \frac{s}{s^2-b^2}ds$$

$$= \left[\log(s-a) - \frac{1}{2}\log(s^2+b^2)\right]_s^\infty$$

$$= \left(\log\frac{s-a}{\sqrt{s^2+b^2}}\right)_s^\infty$$

$$= -\log\frac{\text{s}-\text{a}}{\sqrt{\text{s}^2+\text{b}^2}} = \log\frac{\sqrt{s^2+b^2}}{s-a}$$

3. (c) Find the inverse Laplace transform of $\frac{S}{(S-3)(S^2+4)}$ **[2014 (w) Q. 6 (c)]**

Ans : $L^{-1}\frac{S}{(S-3)(S^2+4)}$

By partial fraction : $\frac{s}{(S-3)(S^2+4)} = \frac{A}{S-3} + \frac{BS+C}{S^2+4}$ -------- (1)

$$\Rightarrow \quad \frac{S}{(S-3)(S^2+4)} = \frac{A(S^2+4)+BS+C(S-3)}{(S-3)(S^2+4)}$$

$$\Rightarrow \quad S = A(S^2+4)+BS+C(S-3)$$

$$\Rightarrow \quad S = AS^2+4A+Bs^2-3BS+CS-3C$$

Putting S - 3 =0

S = 3

3 = A (9+4) + 0

$\Rightarrow$ A = 3/13

Equating the co-eff. of B^2

0 = A +B

$\Rightarrow$ B = -3/12

Equating the co-eff. of S

$1 = -3B + C \qquad \Rightarrow 1 = \frac{9}{13} + C$

$\Rightarrow C = 1 - \frac{9}{13} = \frac{4}{13}$

By putting the vol. of A, B, C in equation (1), we get

$$\frac{S}{(S-3)(S^2+4)} = \frac{3/13}{S-3} + \frac{\frac{-3}{13}S + \frac{4}{13}}{S^2+4}$$

$$= \frac{1}{13}\left[\frac{3}{S-3} + \frac{4-3S}{S^2+4}\right]$$

$$L^{-1}\left(\frac{S}{(S-3)(S^2+4)}\right) = \frac{1}{13}\left[L^{-1}\left(\frac{3}{S-3}\right) + L^{-1}\left(\frac{4-3S}{S^2+4}\right)\right]$$

$$= \frac{1}{13}\left[3e^{3t} + L^{-1}\left(\frac{4}{S^2+4}\right) - 3L^{-1}\left(\frac{S}{S^2+4}\right)\right]$$

$$= \frac{1}{13}\left[3e^{3t} + 2\sin 2t - 3\cos 2t\right]$$

4. (a) Find Laplace transform of f(t) =k where K is a constant and t ≥ **[2015 (w) Q. 3 (a)]**

Ans : $L\{f(t)\} = L(K)$

$= KL(I)$

$= K.\frac{1}{S}$

$= \frac{K}{S}$

(b) Find Laplace transform of f(t) =cosh at. Cos bt **[2015 (w) Q. 3 (b)]**

Ans : $L\{f(t)\} = L\{\cosh at.\cos bt\}$

$$= L\left[\left(\frac{e^{at}+e^{-at}}{2}\right)\cos bt\right]$$

$$= \frac{1}{2}L\left[e^{at}\cos bt + e^{-at}\cos bt\right]$$

$$= \frac{1}{2}L\left[\left(e^{at}\cos bt\right) + L\left(e^{-at}\cos bt\right)\right]$$

$$= \frac{1}{2}\left[\frac{S-a}{(s-a)^2+b^2} + \frac{S+a}{(s+a)^2+b^2}\right]$$

5. (a) Find Inverse Laplace transform of **[2015 (w) Q. 3 (c)]**

$$\cot - 1\left(\frac{s+a}{b}\right)$$

Ans : $Let \quad f(t) = L^{-1}\left[\cot^{-1}\left(\frac{s+a}{b}\right)\right]$

$$\Rightarrow t \quad f(t) = L^{-1}\left[-\frac{d}{ds}\left\{\cot^{-1}\left(\frac{s+a}{b}\right)\right\}\right]$$

$$\Rightarrow t \quad f(t) = L^{-1}\left[\frac{1}{1+\left(\frac{s+a}{b}\right)^2}\frac{1}{b}\right]$$

$$\Rightarrow t \quad f(t) = L^{-1}\left[\frac{b^2}{b^2+(s+a)^2}.\frac{1}{b}\right]$$

$$\Rightarrow t \quad f(t) = L^{-1}\left[\frac{b}{(s+a)^2+b^2}\right]$$

$$\Rightarrow t \quad f(t) = e^{-at}\sin bt$$

$$\Rightarrow \; f(t) = \frac{e^{-at}\sin bt}{t}$$

5. (b) Find the Laplace transform of $\frac{\sin 2t}{t}$ **[2015 (w) Q. 4 (b)]**

Ans : $L\left[\frac{\sin 2t}{t}\right]$

$$L(\sin 2t) = \frac{2}{s^2+4}$$

$$L\left[\frac{\sin 2t}{t}\right]$$

$$= \int_s^\infty \frac{2}{s^2+4}ds$$

$$= \int_s^\infty \frac{2}{s^2+2^2}ds$$

$$= \left[\tan^{-1}\frac{s}{2}\right]_s^\infty$$

$$= \tan^{-1}\infty - \tan^{-1}\frac{s}{2}$$

$$= \frac{\pi}{2} - \tan^{-1}\frac{s}{2}$$

$$= \cot^{-1}\frac{s}{2}$$

(c) Find the Inverse Laplace transform of $\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)}$

Ans : $L^{-1}\left[\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)}\right]$ [2018, 6]

Partial fraction

$$\frac{2s^2-6s+5}{(s-1)(s-2)(s-3)} = \frac{A}{s-1}+\frac{B}{s-2}+\frac{c}{s-3}$$

$$\Rightarrow \quad \frac{2s^2-6s+5}{(s-1)(s-2)(s-3)} = \frac{A(s-2)(s-3)+B(s-1)(s-3)+C(s-1)(s-2)}{(s-1)(s-2)(s-3)}$$

$$\Rightarrow \quad 2s^2 - 6s + 5 = A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2) ----(1)$$

Putting S=1 in equation (1)

$$2(1)^2 - 6\times 1 + 5 = A(1-2)(1-3) + B\times 0 + C + 0$$

$$\Rightarrow \quad 1 \quad = \quad 2A$$

$$\Rightarrow \quad A \quad = \quad \frac{1}{2}$$

Putting S=2

$$\Rightarrow 2\times(2)^2 - 6\times 2 + 5 = A\times 0 + B(2-1)(2-3) + C\times 0$$

$$\Rightarrow \quad 8-12+5 \quad = \quad -B$$

$$\Rightarrow \quad 1 \quad = \quad -B$$

$$\Rightarrow \quad B \quad = \quad -1$$

Putting S=3

$$18-18+5 \quad = \quad C\times 2\times 1$$

$$\Rightarrow \quad C \quad = \quad \frac{5}{2}$$

$$L^{-1}\left[\frac{2S^2 - 6S + 5}{(S-1)(S-2)(S-3)}\right]$$

$$= L^{-1}\left[\frac{A}{S-1} + \frac{B}{S-2} + \frac{C}{S-3}\right]$$

$$= L^{-1}\left[\frac{1/2}{S-1} - \frac{1}{S-2} + \frac{5/2}{S-3}\right]$$

$$= \frac{1}{2}L^{-1}\left(\frac{1}{S-1}\right) - L^{-1}\left(\frac{1}{S-2}\right) + \frac{5}{2}L^{-1}\left(\frac{1}{S-3}\right)$$

$$= \frac{1}{2}e^{t} - e^{2t} + \frac{5}{2}e^{3t}$$

(d) Find the Inverse Laplace transform of $Log\left(\frac{1+s}{s}\right)$ **[2015,3.B]**

Ans : $Let \quad f(t) \quad = L^{-1}\left[Log\left(\frac{1+s}{s}\right)\right]$

$$\Rightarrow t \quad f(t) \quad = L^{-1}\left[-\frac{d}{ds}Log\left(\frac{1+s}{s}\right)\right]$$

$$\Rightarrow t \quad f(t) \quad = L^{-1}\left[\frac{-d}{ds}\{Log(1+s) - Logs\}\right]$$

$$\Rightarrow t \quad f(t) \quad = L^{-1}\left[\frac{-d}{ds}Log(1+s) + \frac{d}{ds}\log s\right]$$

$$\Rightarrow t \quad f(t) \quad = L^{-1}\left[\frac{-1}{1+s} + \frac{1}{s}\right]$$

$$\Rightarrow t \quad f(t) \quad = -e^{-t} + 1$$

$$\Rightarrow \quad f(t) \quad = \frac{1-e^{-t}}{t}$$

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CHAPTER – 5

1 (i) Define Dirichlets conditions for a Fourier expansion of f(x) **[2017(W) Q. 1 (ix)]**

The function f(x) can be expanded as a Fourier series f(x) = $f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty}$ $a_n \cos x + \sum_{n=1}^{\infty} b_n$ sin x in interval $\alpha < x < \alpha + 2\pi$. Where a_o, a_n and b_n are constants provided that

i) F(x) is periodic, single valued and finite.
ii) F(x) has only a finite number of finite discontinuities.
iii) F(x) has only a finite no. of local maxima and minima.

ii) Find the fourier co-efficient a_o for the function $f(x) = e^x$ in $-\pi < x < \pi$.
Let the fourier series of the function **[2014(W) Q. 1 (v)]**

$$f(x) = e^x = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \text{ sinnx.}$$

Where $a_o = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).dx$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} e^x .dx$$

$$= \frac{1}{\pi}\left[e^x\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[e^{\pi} - e^{-\pi}\right]$$

$$= \frac{2}{\pi}.\left[\frac{e^{\pi} - e^{-\pi}}{2}\right] = \frac{2}{\pi}\sin h\pi$$

iii) Find the value of fourier co-efficient a_o 2013(w) Q 1(f)
if $f(x) = x + x^2$ in $(-\pi, \pi)$.

Ans : Let the fourier series of given function
$F(x) = x + x^2$ be,

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \text{ sinnx.}$$

Where $a_o = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).dx$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x + x^2).dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2} + \frac{x^3}{3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\pi^2}{2} + \frac{\pi^3}{3} - \frac{(-\pi)^2}{2} - \frac{(-\pi)^3}{3}\right]$$

$$\Rightarrow a_o = \frac{1}{\pi}.2\frac{\pi^3}{3} = \frac{2\pi^2}{3}$$

iv) Find the fourier co-efficient a_o for obtaining a fourier series for $f(x) = e^{-x}$ in $0 < x < 2\pi$ [2015 (w) Q 1(ix)]

Ans : Given $f(x) = e^{-x}$ in $0 < x < 2\pi$

Let the fourier series be

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

Where $a_o = \frac{1}{\pi}\int_o^{2\pi} f(x).dx = \frac{1}{\pi}\int_o^{2\pi} e^{-x}.dx$

$$= \frac{1}{\pi}\left[\frac{e^{-x}}{-1}\right]_o^{-2\pi}$$

$$= \frac{-1}{\pi}\left[e^{-2x} - e^o\right]$$

$$\Rightarrow a_o = \frac{-1}{\pi}\left[e^{-2x} - 1\right]$$

$$\Rightarrow a_o = \frac{1}{\pi}\left[1 - e^{-2x}\right]$$

iv) Define Eulers formulae.

Ans : The fourier series for the function f(x) in $\alpha < x < \alpha + 2\pi$ is given by 2013-S(1.VI)

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

Where $a_o = \frac{1}{\pi}\int_{\alpha}^{\alpha+2\pi} f(x).dx$

$$a_n = \frac{1}{\pi}\int_{\alpha}^{\alpha+2\pi} f(x).\cos nx.\ dx$$

and $b_n = \frac{1}{\pi}\int_{\alpha}^{\alpha+2\pi} f(x).\sin nx.\ dx$

These values a_o, a_n and b_n are known as Euler's formulae.

ii) Find a Fourier series to represent x^2 in the interval (*-l.l*) **[2013(w) Q2(h)]**

Ans : Since $f(x) = x^2$ is an even function in (*-l.l*)

So $f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \frac{\cos n\pi x}{l}$ ---------(1)

Where $a_o = \frac{2}{l}.\int_o^l x^2.dx$

$$= \frac{2}{l}\left[\frac{x^3}{3}\right]_o^l$$

$$= \frac{2}{l}\left(\frac{l^3}{3} - o\right)$$

$$= \frac{2}{1}.\frac{l^3}{3} = \frac{2l^2}{3}$$

And $a_n = \frac{2}{l}.\int_o^l x^2.\cos \frac{n\pi x}{l}.dx$

$$= \frac{2}{l}\left[x^2 \frac{\sin n\pi x/l}{n\pi/l} - \int\left[2x.\frac{\sin n\pi x/l}{n\pi/l}\right].dx\right]_o^l$$

$$= \frac{2}{l}\left[x^2.\frac{\sin n\pi x/l}{n\pi/l} - 2x\left(\frac{-\cos n\pi x/l}{n^2\pi^2/l^2}\right) + 2\int.\frac{-\cos n\pi x/l}{n^2\pi^2/l^2}.dx\right]_o^l$$

$$= \frac{2}{1}\left[o + 2x\frac{\cos n\pi x/1}{n^2\pi^2/1^2} + 2\left(\frac{-\sin n\pi x/1}{n^3\pi^3/1^3}\right)\right]_o^1$$

$$= \frac{2}{l}.\left[2x\frac{\cos n\pi x/l}{n^2\pi^2/l^2} - o\right]_o^l$$

$$= \frac{2}{l}.\ 2l\frac{\cos n\pi x/l}{n^2\pi^2/l^2} - o$$

$$= 4\cos n\pi \times \frac{l^2}{n^2\pi^2}$$

$$= \frac{4l^2\cos n\pi}{n^2\pi^2}$$

$$\Rightarrow a_n = \frac{4l^2\,(-1)^n}{n^2\pi^2} \qquad \left(\because \cos n\pi = (-1)^n\right)$$

Substituting these values in equation (1) we get

$$x^2 = \frac{l^2}{3} - \frac{4l^2}{\pi^2}\left[\frac{\cos \pi x/l}{1^2} - \frac{\cos 2\pi x/l}{2^2} + - - - - -\right]$$

3. (i) Find the Fourier series of the function

$$f(x) = \begin{cases} 0, & -2 < x < -1 \\ K, & -1 < x < 1 \\ O, & 1 < x < 2 \end{cases} \qquad [2014,Q.6]$$

Solution : We know that

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \frac{\cos n\pi x}{2} + \sum_{n=1}^{\infty} b_n \frac{\sin n\pi x}{2}$$

Now $a_o = \frac{1}{2}\int_{-2}^{2} f(x).dx$

$$= \frac{1}{2}\left[\int_{-2}^{-1} f(x).dx + \int_{-1}^{1} f(x).dx + \int_{1}^{2} f(x).dx\right]$$

$$= \frac{1}{2}\left[\int_{-2}^{-1} o.dx + \int_{-1}^{1} k.dx + \int_{1}^{2} o.dx\right]$$

$$= \frac{1}{2}\left[\int_{-1}^{1} k.\,dx = \frac{1}{2}k[x]_{-1}^{1} = \frac{k}{2}(1+1)\right]$$

$$= \frac{k}{2}.2 = k$$

And $$a_n = \frac{1}{2} + \int_{-2}^{2} f(x).\ \cos\frac{n\pi x}{2}.dx$$

$$= \frac{1}{2}\left[\int_{-2}^{-1} f(x).\cos\frac{n\pi x}{2}.dx + \int_{-1}^{1} f(x).\cos\frac{n\pi x}{2}.dx + \int_{1}^{2} f(x).\cos\frac{n\pi x}{2}.dx\right]$$

$$= \frac{1}{2} + \left[0 + \int_{-1}^{1} k.\ \cos\frac{n\pi x}{2}.dx + 0\right]$$

$$= \frac{1}{2}\int_{-1}^{1} k.\ \cos\frac{n\pi x}{2}.dx = \frac{k}{2}\int_{-1}^{1}.\ \cos\frac{n\pi x}{2}.dx$$

$$= \frac{k}{2}\left[\sin\frac{n\pi x/2}{n\pi/2}\right]_{-1}^{1}$$

$$= \frac{k}{2}.\frac{2}{n\pi}.[\sin n\pi/2 - \sin(-n\pi/2)\ ]$$

$$= \frac{k}{n\pi}.\left[\sin\frac{n\pi}{2} + \sin\frac{n\pi}{2})\right]$$

$$\Rightarrow a_n = \frac{2k}{n\pi}.\sin\frac{n\pi}{2}$$

$$\Rightarrow a_n = \begin{cases} 2k/n\pi \text{ when n is odd} \\ o\ ,\ \text{when n is even.}\end{cases}$$

And $$b_n = \frac{1}{2}\int_{-2}^{2} f(x).\sin\frac{n\pi x}{2}.dx$$

$$= \frac{1}{2}\left[\int_{-2}^{-1} f(x).\ \sin\frac{n\pi x}{2}\ .\ dx + \int_{-1}^{1} f(x).\sin\frac{n\pi x}{2}\ .\ dx + \int_{1}^{2} f(x).\sin\frac{n\pi x}{2}\ .\ dx\right]$$

$$= \frac{1}{2} + \int_{-1}^{1} k.\ \sin\frac{n\pi x}{2}.dx$$

$$= \frac{k}{2}.k\left[\frac{-\cos n\pi x/2}{n\pi/2}\right]_{-1}^{1}$$

$$= \frac{k}{2}.\frac{2}{n\pi}[-\cos n\pi x/2 + \cos n\pi/2\ ] = 0$$

$$b_n = 0$$

Hence $$f(x) = \frac{k}{2} + \frac{2k}{\pi}\left[\cos\frac{\pi x}{2} + \frac{1}{3}\cos\frac{3\pi x}{2} + -----\right]$$

ii) Find the Fourier series of the following function 2016(w) Q.3

$$f(x)=\begin{cases} x^2, 0 \le x \le \pi \\ -x^2, -\pi, x \le 0 \end{cases}$$

Solution : Here the Fourier series

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos nx+\sum_{n=1}^{\infty} b_n \sin nx$$

Where, $$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x).dx=\frac{1}{\pi}\left[\int_{-\pi}^{0}-x^2.dx+\int_{0}^{\pi} x^2 dx\right]$$

$$=\frac{1}{\pi}\left\{\left[\frac{-x^3}{3}\right]_{-\pi}^{o}+\left[\frac{x^3}{3}\right]_{o}^{\pi}\right\}$$

$$\Rightarrow \quad \mathrm{a_o}=\frac{1}{\pi}\left\{0+\frac{(-\pi)^3}{3}+\frac{\pi^3}{3}-0\right\}$$

$$\Rightarrow \quad \mathrm{a_o}=\frac{1}{\pi}\left(\frac{-\pi^3}{3}+\frac{\pi^3}{3}\right)$$

$$\Rightarrow \quad \mathrm{a_o}=0$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x).\cos nx.dx$$

$$=\frac{1}{\pi}\left\{\int_{-\pi}^{o}-x^2.\cos nx.dx+\int_{o}^{\pi} x^2.\cos nx.dx\right\}$$

$$=\frac{1}{\pi}\left\{\left[-x^2.\frac{\sin nx}{n}-\int(-2x).\frac{\sin nx}{n}.dx\right]_{-\pi}^{o}+\left[x^2.\frac{\sin nx}{2}-\int 2x.\frac{\sin nx}{n}.dx\right]_{o}^{\pi}\right.$$

$$=\frac{1}{\pi}\left\{\left[-x^2.\frac{\sin nx}{n}+2x.\left(\frac{-\cos nx}{n^2}\right)-\int -2.\frac{\cos nx}{n^2}.dx\right]_{-\pi}^{o}+\right.$$

$$\left[x^2.\frac{\sin nx}{2}-2x\left(\frac{\cos nx}{n^2}\right)+\int 2.\frac{-\cos nx}{n^2}.dx\right]_{o}^{\pi}$$

$$=\frac{1}{\pi}\left\{\left[0-2x.\frac{\cos nx}{n^2}+2.\left(\frac{-\sin nx}{n^3}\right)\right]_{-\pi}^{o}+\left[0+2x.\frac{\cos nx}{n^2}+2\left(\frac{-\sin nx}{n^3}\right)\right]_{o}^{\pi}\right\}$$

$$=\frac{1}{\pi}\left\{\left[-2x.\frac{\cos nx}{n^2}\right]_{-\pi}^{o}+\left[2x\frac{\cos nx}{n^2}\right]_{o}^{\pi}\right\} \qquad (\because \sin \mathrm{nx}=0)$$

$$=\frac{1}{\pi}\left[0+2\pi.\frac{\cos n(-\pi)}{n^2}+2\pi\frac{\cos nx}{n^2}-0\right]$$

$$=\frac{1}{\pi}\left[-2\pi.\frac{\cos n\pi}{n^2}+2\pi\frac{\cos n\pi}{n^2}\right]$$

$$\Rightarrow a_n=0.$$

Again $b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x).\sin nx.dx$

$$=\frac{1}{\pi}\left\{\int_{-\pi}^{o}-x^2.\sin nx.dx+\int_{o}^{\pi}x^2.\sin nx.dx\right\}$$

$$=\frac{1}{\pi}\left\{\left[-x^2.\left(\frac{-\cos nx}{n}\right)-\int(-2x).\frac{-\cos nx}{n}.dx\right]_{-\pi}^{o}+\left[x^2.\left(\frac{-\cos nx}{n}\right)-\int 2x.\left(\frac{-\cos nx}{n}\right).dx\right]_{o}^{\pi}\right\}$$

$$=\frac{1}{\pi}\left\{\left[x^2.\frac{\cos nx}{n}+2x\left(\frac{-\sin nx}{n^2}\right)-\int 2.\frac{-\sin nx}{n^2}.dx\right]_{-\pi}^{o}+\left[-x^2.\frac{\cos nx}{n}+2x\left(\frac{\sin nx}{n^2}\right)-\int 2.\frac{\sin nx}{n^2}.dx\right]_{o}^{\pi}\right\}$$

$$=\frac{1}{\pi}\left\{\left[\frac{x^2\cos nx}{n}+0+2\left(\frac{-\cos nx}{n^3}\right)\right]_{-\pi}^{o}+\left[\frac{-x^2\cos nx}{n}+0+2\frac{-\cos nx}{n^3}\right]_{0}^{\pi}\right\}$$

$$=\frac{1}{\pi}\left\{\left[x^2\frac{\cos nx}{n}-2\frac{\cos nx}{n^3}\right]_{-\pi}^{o}+\left[-x^2\frac{\cos nx}{n}+2\frac{\cos nx}{n^3}\right]_{0}^{\pi}\right\}$$

$$=\frac{1}{\pi}\left\{\left[0-\frac{2.1}{n^3}-\left(\pi^2\frac{\cos n\pi}{n}-2\frac{\cos n(-\pi)}{n^3}\right)\right]+\left(-\pi^2\frac{\cos nx}{n}+2\frac{\cos nx}{n^3}\right)-\left(0+\frac{2}{n^3}\right)\right\}$$

$$=\frac{1}{\pi}\left[\frac{-2}{n^3}-\pi^2\frac{\cos n\pi}{n}+2\frac{\cos n\pi}{n^3}-\pi^2\frac{\cos n\pi}{n}+2\frac{\cos n\pi}{n^3}-\frac{2}{n^3}\right]$$

$$=\frac{1}{\pi}\left[\frac{-4}{n^3}-\frac{2}{n}\pi^2\cos n\pi+\frac{4}{n^3}\cos n\pi\right]$$

$$=\frac{1}{\pi}\left[\frac{-4}{n^3}(1-\cos n\pi)-\frac{2\pi^2}{n}.\cos n\pi\right]$$

$$=\frac{2}{\pi}\left[-\frac{2}{n^3}(1-\cos n\pi)-\frac{\pi^2}{n}.\cos n\pi\right]$$

$$\Rightarrow b_n=\frac{2}{\pi}\left[\frac{2}{n^3}(1-(-1)^n-\frac{\pi^2}{n}.(-1)^n\right]$$

$$\therefore f(x)=\frac{-2}{\pi}\sum\left[\frac{2}{n^3}(1-(-1)^n-\frac{\pi^2}{n}.(-1)^n\right].\sin nx.$$

4. (i) Find the Fourier co-efficient a_o for obtaining of Fourier series for $f(x) = e^{-x}$, $0<x<2\pi$

Solution : **[2014 2(a)]**

$$a_o=\frac{1}{\pi}\int_0^{2\pi}e^{-x}dx$$

$$=\frac{1}{\pi}\left[-e^{-x}\right]_o^{2\pi}$$

$$=\frac{1}{\pi}\left[-e^{-2\pi}+e^o\right]$$

$$=\frac{1}{\pi}\left(1-e^{-2\pi}\right)$$

ii) Find the Fourier sine series to represent f(x) = x in 0 <x<π [2016W,2.B]

Solution :

F(x) = x is an odd Function

Half range sine series

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_b = \frac{2}{\pi}\int_o^{\pi} x \sin nx \, dx$$

$$= \frac{2}{\pi}\left[-x\frac{\cos nx}{n} + \frac{\sin nx}{n^2}\right]_o^{\pi}$$

$$= \frac{2}{\pi}\left[-\pi\frac{\cos n\pi}{n} + \frac{\sin n\pi}{n^2} + 0\frac{-\sin o}{n^2}\right]_o^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{-\pi}{n}(-1)^n + 0 + 0 - 0\right]$$

$$= -2\frac{(-1)^n}{n}$$

$$\text{f(x)} = \sum_{n=1}^{\infty} -2 \frac{(-1)^n}{n}\sin nx$$

$$= -2\sum_{n=1}^{\infty}\frac{(-1)^n}{n} \sin nx$$

$$\text{f(x)} = -2 \left[\frac{-1}{1}\sin x + \frac{1}{2}\sin 2x - \frac{1}{3}\sin 3x + \frac{1}{4}\sin 4x.........\right]$$

$$= 2 \left[\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + ...\right]$$

iii) Express f(x) = |x| as a Fourier series in = -π< x <π

Ans : f(x) = |x| (- π< x <π)

f(x) is an even function **[2014 . Q. 3(c)]**

$$\text{f(x)} = \begin{cases} -x & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$$

$$\text{f(x)} = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_o = \frac{2}{\pi}\int_o^{\pi} x \, dx$$

$$= \frac{2}{\pi}\left[\frac{x^2}{2}\right]_o^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right] = \pi$$

$$a_n = \frac{2}{\pi}\int_o^{\pi} x \cos nx \, dx$$

$$= \frac{2}{\pi}\left[x\frac{\sin nx}{n}+\frac{\cos nx}{n^2}\right]_o^\pi$$

$$= \frac{2}{\pi}\left[\pi\frac{\sin n\pi}{n}+\frac{\cos n\pi}{n^2}-0-\frac{\cos o}{n^2}\right]$$

$$a_n = \frac{2}{\pi}\left[0+\frac{(-1)^n}{n^2}+\frac{1}{n^2}\right]$$

$$= \frac{2}{\pi n^2}\left[(-1)^n-1\right]$$

$$f(x) = \frac{\pi}{2}+\sum_{n=1}^{\infty}\frac{2}{\pi n^2}\left[(-1)^n-1\right]\cos nx$$

$$= \frac{\pi}{2}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\left[(-1)^n-1\right]}{n^2}\cos nx$$

$$= \frac{\pi}{2}+\frac{2}{\pi}\left[\frac{-2}{1^2}\cos x+0-\frac{2}{3^2}\cos 3x+0\frac{-2}{5^2}\cos 5x\right]$$

$$f(x)= \frac{\pi}{2}-\frac{4}{\pi}\left[\frac{\cos x}{1^2}+\frac{\cos 3x}{3^2}+\frac{\cos 5x}{5^2}+----\right]$$

iv) Find a Fourier sine series for the function

$f(x) = e^{ax}$ for $0 < x < \pi$

Solution: $f(x) = e^{ax}$

Fourier sine series. **[2015 . Q. 5(c)]**

$$f(x)= \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{\pi}\int_o^\pi e^{ax}\sin nx \ dx$$

$$= \frac{2}{\pi}\left[\frac{e^{ax}}{a^2+n^2}(a\sin nx - n\cos nx)\right]_o^\pi$$

$$= \frac{2}{\pi}\left[\frac{e^{a\pi}}{a^2+n^2}(a\sin n\pi - n\cos n\pi)-\frac{e^o}{a^2+n^2}(o-n.\cos 0)\right]$$

$$= \frac{2}{\pi}\left[\frac{e^{a\pi}}{a^2+n^2}\left\{-n(-1)^n\right\}+\frac{n}{a^2+n^2}\right]$$

$$= \frac{2n}{\pi\left(a^2+n^2\right)}\left[1-(-1)^n e^{a\pi}\right]$$

$$f(x)=\sum_{n=1}^{\infty} b_n \sin nx$$

$$=\sum_{n=1}^{\infty}\frac{2n}{\pi\left(a^2+n^2\right)}\left\{1-(-1)^n e^{a\pi}\right\}\sin nx$$

$$= \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{n}{a^2+n^2}\left[1-(-1)^n e^{a\pi}\right]\sin x \quad = \frac{2}{\pi}\left[\frac{2e^{a\pi}}{a^2+1}\sin x+\frac{2}{a^2+4}(1-e^{a\pi})\sin 2x+......\right]$$

4. (v) Find a Fourier series of the function

$$f(x)=\begin{cases}-1 & for \quad -\pi<x<0 \\ 1 & for \quad 0<x<\pi\end{cases}$$

[2015 (w). Q. 5 (b)]

Solution : $$f(x)=\frac{a_o}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$$

$$a_o = \frac{1}{\pi}\left[\int_{-\pi}^{o}-1\,dx+\int_{o}^{\pi}1.\,dx\right]$$

$$= \frac{1}{\pi}\left[[\text{-x}]_{-\pi}^{o}+[\text{x}]_{o}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[o-\pi+\pi-o\right] \quad =0$$

$$a_n=\frac{1}{\pi}+\int_{-\pi}^{o}-\cos nxdx+\int_{o}^{\pi}\cos nx\,dx$$

$$=\frac{1}{\pi}\left[\left[-\frac{\sin nx}{n}\right]_{-\pi}^{o}+\left[-\frac{\sin nx}{n}\right]_{o}^{\pi}\right]$$

$$=\frac{1}{\pi}\left[-o\frac{-\sin n\pi}{n}+\frac{\sin n\pi}{n}-o\right]=o$$

$$b_n=\frac{1}{\pi}+\left[\int_{-\pi}^{o}-\sin nxdx+\int_{o}^{\pi}\sin nx\,dx\right]$$

$$=\frac{1}{\pi}\left[\left[\frac{\cos nx}{n}\right]_{-\pi}^{o}+\left[-\frac{\cos nx}{n}\right]_{o}^{\pi}\right]$$

$$=\frac{1}{\pi}\left[\frac{1}{n}-\frac{\cos n\pi}{n}-\frac{\cos n\pi}{n}+\frac{1}{n}\right]$$

$$=\frac{1}{\pi}\left[\frac{2}{n}-\frac{2(-1)^n}{n}\right]$$

$$=\frac{1}{\pi n}\left[1-(-1)^n\right]$$

$$f(x)=\frac{a_o}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$$

$$=0+0+\sum_{n=1}^{\infty}\frac{2[1-(-1)^n]}{\pi n}\sin nx$$

$$=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n]}{n}\sin nx$$

$$=\frac{2}{\pi}\left[\frac{2}{1}\sin x+0+\frac{2}{3}\sin 3x+0+\frac{2}{5}\sin 5x+....\right]$$

$$=\frac{4}{\pi}\left[\frac{\sin x}{1}+\frac{\sin 3x}{3}+\frac{\sin 5x}{5}+....\right]$$

CHAPTER-6

1. (i) Find the 1st approx. to the root of the equation : x^3-4x-9 = 0 using Bisection Method.

Ans : $f(x) = x^3-4x-9=0$ [2018, 1.V]

$f(o) = -9 < 0$

$f(1) = 1- 4 - 9 = -12 < 0$

$f(2) = 8 - 8 - 9 = - 9 < 0$

$f(3) = 27 - 12 - 9 = 6 > 0$

So, Root lies between 2 & 3.

First approx. to the root is

$$x_1 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$$

(ii) Write down the working procedure of Newton Raphson Method

Ans : Consider $f(x) = 0$

(i) Find the initial & final point such that the root of $f(x)$ lies between that pt.

If f(a) f(b)<0.

then f(x) lies between a & b.

choose $x_o = \dfrac{a+b}{2}$

(ii) Find the 1st approx. by the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

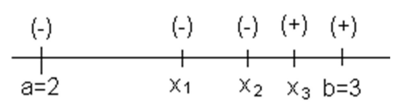
by putting n = 0, 1, 2…….

2 (a) Find a root of the equation

x^3- 4x-9= 0 [2013,5.B]

using Bisection Method in four stages.

Ans :*let* $f(x) = x^3- 4x-9$



$f(o) = -9 < 0$

$f(1) = 1- 4 - 9 = -12 < 0$

$f(2) = 8 - 8 - 9 = - 9 < 0$

$f(3) = 27 - 12 - 9 = 6 > 0$

So, Root lies between 2 & 3.

First approx is

$$x_1 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$$

$$\begin{aligned} f(x_1) &= x^3 - 4x - 9 \\ &= (2.5)^3 - 4(2.5) - 9 \\ &= 15.625 - 10 - 9 \\ &= -3.375(-ve) \end{aligned}$$

$\Rightarrow$ The root lies between x_1& 3

$$x_2 = \frac{x_1+3}{2} = \frac{2.5+3}{2} = \frac{5.5}{2} = 2.75$$

$$\begin{aligned} f(x_2) &= x_2^{\,3} - 4(x_2) - 9 \\ &= (2.75)^3 - 4(2.75) - 9 \\ &= 15.625 - 10 - 9 \\ &= -1.203125(-ve) \end{aligned}$$

So, the root lies between x_2& 3.

3^{rd}approx is

$$x_3 = \frac{x_2+3}{2} = \frac{2.75+3}{2} = 2.875$$

$$f(x_3) = x^3 - 4(x_3) - 9$$

$$= (2.875)^3 - 4(2.875) - 9$$

$$= 3.263(+ve)$$

The root lies between x_3&x_2.

4^{th}approx is

$$(x_4) = \frac{x_3+x_2}{2} = \frac{2.875+2.75}{2}$$

$$= 2.8125$$

(b) Find by Newton's Method, a root of the equation $x^3 - 3x + 1 = 0$ correct to 3 decimal places. **[2013 (w) Q. 6]**

Ans : $f(x) = x^3 - 3x + 1 = 0$

$f(1) = 1 - 3 + 1 = -1$

$f(2) = 2^3 - 3\text{x}2 + 1 = 3$

The root lies between 1 & 2

$$let\ x_o = \frac{1+2}{2} = 1.5$$

$$f(x_o) = x_o^3 - 3x_o + 1 = (1.5)^3 - 3(1.5) + 1$$

$$= -0.125$$

$$f'(x_o) = 3x^2 - 3 \quad , \qquad f'(1.5) = 3(1.5)^2 - 3 = 3.75$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Putting $n = o$ $\quad x_1 = x_o - \frac{f(x_o)}{f'(x_o)}$

$$= 1.5 + \frac{0.125}{3.75} = 1.533$$

Putting $n = 1$ $\quad x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$$f(x_1) = x_1^3 - 3x_1 + 1$$

$$= (1.533)^3 - 3(1.533) + 1$$

$$= 0.0036$$

$$f'(x_1) = 3x_1^2 - 3$$

$$= 3 \times (1.533)^2 - 3$$

$$= 4.050$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.533 - \frac{0.003}{4.050}$$

$$= 1.532$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$f(x_2) = x^3 - 3x + 1$$

$$= (1.532)^3 - 3(1.532) + 1$$
$$= -0.003592$$
$$f(x_2) = 3x_2^2 - 3$$
$$= 3 \times (1.532)^2 - 3$$
$$= 4.041$$
$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= 1.532 + \frac{0.003592}{4.041}$$
$$= 1.532 + 0.0000889$$
$$= 1.53208$$

So the approx root correct upto 3 decimal places is 1.532.

c) Find the cube root of 41 using Newton_Raphson Method. 2014W,Q.6

Solution :

$$Let \quad x = \sqrt[3]{41}$$
$$\Rightarrow \quad x^3 = 41$$
$$\Rightarrow \quad x^3 - 41 = 0$$
$$f(x) = x^3 - 41 \qquad f'(x) = 3x^2$$
$$f(3) = (3)^3 - 41 = 14(-ve)$$
$$f(4) = (4)^3 - 41 = 23(+ve)$$

The root lies between 3 and 4

$$Let \quad x_o = 3$$

1st approx is

$$x_1 = x_o - \frac{f(x_o)}{f'(x_o)}$$
$$= x_o - \frac{(x_o)^3 - 41}{3x_o^2}$$
$$= 3 - \frac{(3)^3 - 41}{3 \times (3)^2}$$
$$= 3.5185185$$
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= x_1 - \frac{(x_1)^3 - 41}{3x_1^2}$$
$$= 3.5185185 - \frac{(3.5185185)^3 - 41}{3 \times (3.5185185)^2}$$
$$= 3.4496125$$
$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= x_2 - \frac{(x_2)^3 - 41}{3(x_2)^2}$$

$$= 3.4496125 - \frac{(3.4496125)^3 - 41}{3 \times (3.4496125)^2}$$

$$= 3.448217805$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= x_3 - \frac{(x_3)^3 - 41}{3 \times (x_3)^2}$$

$$= 3.4482178 - \frac{(3.4482178)^3 - 41}{3 \times (3.4482178)^2}$$

$$= 3.448217824$$

Hence the approximate root of the equation is 3.44821724.

***

CHAPTER-7

1. (i) Evaluate Δ(x+cosx), the interval of differencing being unity

Now $\Delta\,(x + \cos x) = [x + 1 + \cos\ (x+1)] - (x + \cos x)$ **[2013 (w) Q. 1(g)]**

$$= x + 1 + \cos\ (x+1) - x - \cos x$$

$$= 1 + [\cos x(x+1) - \cos x]$$

$$= 1 + 2\sin\frac{x+1+x}{2}.\sin x\frac{-(x+1)}{2}$$

$$= 1 + 2\sin\frac{2x+1}{2}.\sin\left(\tfrac{1}{2}\right)$$

$$= 1 + 2\sin\left(x + \tfrac{1}{2}\right)\left(-\sin\tfrac{1}{2}\right)$$

$$\Rightarrow \Delta(x + \cos x) = 1 - 2\sin\left(x + \tfrac{1}{2}\right).\sin\left(\tfrac{1}{2}\right)$$

(ii) Prove that Δ = E-1 **[2018 (w) Q. 1(j)]**

Now $\Delta f(x) = f(x) + h) - f(x)$

$= E\, f(x) - f(x)$

$= (E-1)\, f(x)$

$\Rightarrow \Delta = E-1$ (Proved)

(iii) State Lagrange's interpolation formula for unequal intervals. **[2013 (w),2018, 1(i)]**

$$f(x) - y = \frac{(x-x_1)(x-x_2)(x-x_3).......(x-x_n)}{(x_o-x_1)(x_o-x_2).............(x_o-x_n)} +$$

$$\frac{(x-x_o)(x-x_2).......(x-x_n)}{(x_1-x_o)(x_1-x_2).............(x_1-x_n)} + ------- +$$

$$\frac{(x-x_o)(x-x_1)(x-x_2).......(x-x_{n-1})}{(x_n-x_o)(x_n-x_1).............(x_n-x_{n-1})}$$

This formula is called Lagrange's interpolation formula

(iv) So that $\Delta = E\nabla = \nabla E$ $\nabla f(x) = f(x) - f(x-h)$

$$\Rightarrow E\nabla f(x) = E(f(x) - f(x-h))$$

$$= Ef(x) - Ef(x-h)$$

$= f(x+h) - f(x-h+h)$

$= f(x+h) - f(x)$

$\Rightarrow E\nabla f(x) = \Delta f(x)$

$\Rightarrow E\nabla = \Delta$

Again $E\nabla = \nabla Ef(x) = \nabla(f(x+h)) = f(x+h) - f(x)$

$\Rightarrow \nabla Ef(x) = \nabla f(x)$

$\Rightarrow \nabla E = \nabla$

Hence $\nabla = E\nabla = \nabla E(proved)$

v) Define Numerical integration and state Simpson's one-third rule.

Ans : Numerical integration is the process of computing the rule of a definite integral from the tabulated values of the integrands. **[2013(w) Q. 1 (j)]**

$$\int_{x_o}^{x_o+nh} f(x).dx = \frac{h}{3}[(y_o + y_n) + 4(y_1 + y_3 + ---- + y_{n-1}) + 2(y_2 + y_4 --- + y_{n-2})]$$

This is known as the Simpson's one –third rule or simply Simpson's rule.

(vi) Show that $E^{-1}=1-\nabla$ **[2017(w) Q. 1 (vii)]**

$\nabla f(x) = f(x) - f(x-h)$

$= f(x) - E^{-1}f(x)$

$= (1-E^{-1})f(x)$

$\Rightarrow \nabla = 1 - E^{-1}$

$\Rightarrow \nabla - 1 = -E^{-1}$ $\Rightarrow 1 - \nabla = E^{-1}(proved)$

(vii) State Newton's Backward interpolation formula for unequal intervals.

$$P(x) = f_n + \frac{(x-x_n)}{\angle 1\, h}\nabla f_n + \frac{(x-x_n)(x-x_{n-1})}{\angle 2.h^2}\nabla^2 f_n$$ **[2017 (w) Q. 1(x)]**

$$+ ------ + \frac{(x-x_n)(x-x_{n-1}) ----- (x-x_1)}{\angle n.h^2}\nabla^n f_n$$

This is known as Newton's Backward interpolation formula for equal intervals.

2. (i) By forming a difference table, find the missing values in the following table. Assuming that the fourth differences are equal to zero.

x :	0	5	10	15	20	25
y :	6	10	–	17	-	21

Solution : Let the missing value be y_1 and y_2.

The following is the difference table.

x	y	$\Delta f(x)$	$x^2 f(x)$	$x^3 f(x)$	$\Delta^4 f(x)$
0	6				
5	10	4			
10	y_1	y_1-10	y_1-14	41-3y_1	y_2+6y_1-102
15	17	17- y_1	27-2 y_1	y_2 +3y_1-61	143- 4y_2-4y_2
20	y_2	y_2-17	y_2+ y_1-34	82-3y_2-y_1	
25	31	31- y_2	48 -2 y_2		

Equating the 4th difference to Zero, we get

$y_2 + 6y_1 - 102 = 0$

$143 - 4y_2 - 4y_1 = 0$

$$\Rightarrow 4\ [6y_1 - y_2 - 102] = 0 -------(1)$$

$$-4y_1 - 4y_2 + 143 = 0 ----(2)$$

(+)

$$20y_1 + 0 - 265 = 0$$

$$\Rightarrow 20y_1 = 265$$

$$\Rightarrow y_1 = \frac{265}{20} = \frac{53}{4} = 13.25$$

$$\therefore equation\ (2) \Rightarrow -4 - 13.25 - 4y_2 + 143 = 0$$

$$\Rightarrow -4y_2 - 53 + 143 = 0$$

$$\Rightarrow -4y_2 = -90$$

$$\Rightarrow y_2 = \frac{90}{4} = 22.5$$

$$\therefore y_1 = 13.25 \quad and \quad y_2 = 22.5$$

(ii) Using Newton's forward formula, find the value of $f(1.6)$ if **[2017(w) Q.2(h)]**

$x =$	1	1.4	1.8	2.2
$f(x) =$	3.49	4.82	8.96	6.5

Ans : The difference table is

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	3.49			
		1.3		
1.4	4.82		-0.19	
		1.14		-0.41
1.8	5.96		-0.6	
		0.54		
2.2	6.5			

Using Newton's forward interpolation formula we have,

$$f(x_o + uh) = f(x_o) + \frac{u}{\angle 1}\Delta f(x_o) + \frac{u(u-1)}{\angle 2}\Delta^2 f(x_o) + \frac{u(u-1)(u-2)}{\angle 3}\Delta^3 f(x_o)$$

Here $x_o + uh = 1.6$

$$\Rightarrow 1 + u(0.4) = 1.6 (h = 0.4)$$

$$\Rightarrow u.(0.4) = 1.6 - 1 = 0.6$$

$$\Rightarrow u = \frac{0.6}{0.4} = 1.5$$

$$\therefore f(1.6) = 3.49 + (1.5).(1.33) + \frac{(1.5)(1.5-1)}{\angle 2}(-0.191) + \frac{(1.5)(1.5-1)(1.5-2)}{\angle 3} \times (-0.41)$$

$$= 3.49 + 1.995 + \frac{0.5.1.5}{2}.(-0.19) + \frac{(1.5).(0.5).(-0.5)}{6}.(-0.41)$$

$$= 5.485 - 0.07125 + 0.025625 = 5.439375 = 5.44$$

3. (i) Evaluate $\Delta \tan^{-1}\left(\frac{n-1}{n}\right)$ **[2014 (w) Q. 6(a)]**

Ans : $\Delta \tan^{-1}\left(\frac{n-1}{n}\right)$

$$= \tan^{-1}\left(\frac{n+h-1}{n+h}\right) = \tan^{-1}\left(\frac{n-1}{n}\right)$$

$$= \tan^{-1}\left\{\frac{\frac{n+h-1}{n+h}-\frac{n-1}{n}}{1+\frac{n+h-1}{n+h}.\frac{n-1}{n}}\right\}$$

$$= \tan^{-1}\left\{\frac{n^2+nh-n-n^2-nh+n+h/n(n+h)}{\frac{n^2+nh+n^2-n+hn-h-n+1}{n(n+h)}}\right\}$$

$$= \tan^{-1}\left\{\frac{h}{n(n+h)}\Big/\frac{2n^2+2nh-2n-h+1}{n(n+h)}\right\}$$

$$= \tan^{-1}\left\{\frac{h}{2n^2+2nh-2n-h+1}\right\}$$

(ii) Evaluate $\int_0^4 e^x dx$ **[2014 (w) Q. 4(b)]**

Using Simpson's 1/3rd rule, taking h =1

Ans : Divide the interval (0, 4) into 4 parts each of width h =.1

The values of $f(x) = e^x$ are given below.

x	0	1	2	3	4
$f(x)$	e^o =1 y_o	e^1 =2.72 Y_1	e^2 =7.39 Y_2	e^3 =20.09 Y_3	e^4 =54.6 Y_4

By Simpson's 1/3rd rule.

$$\int_0^4 e^x dx = \frac{h}{3}\left[(y_o+y_4)+4(y_1+y_3)+2y_2\right]$$

$$= \frac{1}{3}\left[1+54.6+4(2.72+20.09)+2\times 7.39\right]$$

$$= \frac{1}{3}\left[55.6+4(22.81)+14.78\right]$$

$$= \frac{1}{3}\left[161.62\right] = 53.87$$

4. (i) Use lagrange's interpolation formula to find the value of y when x =10 if the values of x and y are given.

x : 5 6 9 11

y : 12 13 14 16 **[2015(w) Q. (6)]**

Here $x_o = 5$, $x_1 = 6$, $x_2 = 9$, $x_3 = 11$

and $y_o = 12$, $y_1 = 13$, $y_2 = 14$, $y_3 = 16$

putting x = 10 and substituting the above value in Lagrange's formula we get

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_o-x_1)(x_o-x_2)(x_o-x_3)}.f(x_o) + \frac{(x-x_o)(x-x_2)(x-x_3)}{(x_1-x_o)(x_1-x_2)(x_1-x_3)}.f(x_1)$$

$$+ \frac{(x-x_o)(x-x_1)(x-x_3)}{(x_2-x_o)(x_2-x_1)(x_2-x_3)}.f(x_2) + \frac{(x-x_o)(x-x_1)(x-x_2)}{(x_3-x_o)(x_3-x_1)(x_3-x_2)}.f(x_3)$$

$$\Rightarrow y = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)}.12 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)}.13$$

$$+\frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)}.14+\frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)}.16$$

$$=\frac{4.1(-1)}{(-1).(-4).(-6)}.12+\frac{5.1.(-1)}{1.(-3).(-5)}.13+\frac{5.4.(-1)}{4.3.(-2)}.14+\frac{5.4.1}{6.5.2}.16$$

$$=\frac{-48}{-24}+\left(\frac{-65}{15}\right)+\left(\frac{-280}{24}\right)+\frac{320}{60}$$

$$=2-4.33+11.66+5.33$$

$$=14.66$$

$$y=f(x)=f(10)=14.66$$

(ii) Evaluate $\int_o^1 \frac{dx}{1+x^2} u\sin g$. **[2018(w) Q.2 g]**

i) Trapezoidal rule

ii) Sympson's one-third rule, taking $h=\frac{1}{4}$

Hence compute approximate value of π in each case.

Ans : Here the given integral is

$$\int_o^1 \frac{1}{1+x^2}.dx$$

Divide the range into four equal parts each of width $h=\frac{1}{4}$

Clearly, $h=\frac{1}{4}\therefore n=\frac{x_n-x_o}{h}=\frac{1-0}{1/4}=4$

Hence n =4

x	$y=\frac{1}{1+x^2}$
$x=0$	$y_o=1$
$x_1=\frac{1}{4}$	$y_1=\frac{1}{1+\left(\frac{1}{4}\right)^2}=\frac{16}{17}$
$x_2=\frac{2}{4}=\frac{1}{2}$	$y_3=\frac{1}{1+\left(\frac{1}{2}\right)^2}=\frac{4}{5}$
$x_3=\frac{3}{4}$	$y_4=\frac{1}{1+\left(\frac{3}{4}\right)^2}=\frac{16}{25}$
$x_4=\frac{4}{4}=1$	

	$y_5 = \frac{1}{1+1^2} = \frac{1}{2}$

i) So by Trapezoidal rule.

$$\int_{x_o}^{x_o+nh} y.dx = \frac{h}{2}\left[(y_o + y_n) + 2(y_1 + y_2 + ------ + y_{n-1})\right]$$

$$\therefore \int_{x_o}^{1} \frac{1}{1+x^2}.dx = \frac{h}{2}\left[(y_o + y_4) + 2(y_1 + y_2 + y_3)\right]$$

$$= \frac{h}{2}\left[\left(1+\frac{1}{2}\right) + 2\left(\frac{16}{17} + \frac{4}{5} + \frac{16}{25}\right)\right]$$

$$= \frac{1}{4}.\frac{1}{2}\left[\frac{3}{2} + 2\left(\frac{16}{17} + \frac{4}{5} + \frac{16}{25}\right)\right]$$

$$\cong 0.7828$$

ii) Simpson's 1/3 rule

$$\int_{x_o}^{x_o+nh} y.dx = \frac{h}{3}\left[(y_o + y_n) + 4(y_1 + y_3) + 2y_2)\right]$$

$$= \frac{1}{4.3}\left[\left(1+\frac{1}{2}\right) + 4\left(\frac{16}{17} + \frac{16}{25}\right) + 2.\frac{4}{5}\right]$$

$$= \frac{1}{12}.\left[1.5000 + 4\times 1.5811 + 1.6\right]$$

$$= \frac{1}{12}\times 9.4244 \qquad = 0.7854$$

Hence simpson's 1/3 rule for four ordinates (n=4), $\pi = 4 \times 0.7854 = 3.1416$

5. (i) Evaluate $\Delta\left(\tan^{-1} x\right)$ **[2015 Q. 6(a)]**

Solution : $\Delta\left(\tan^{-1} x\right)$

$$= \tan^{-1} x(x+h) - \tan^{-1} x$$

$$= \tan^{-1}\left[\frac{x+h-x}{1+(x+h)x}\right]$$

$$= \tan^{-1}\left(\frac{h}{1+x^2+hx}\right) \qquad \text{(h=1)}$$

$$= \tan^{-1}\left(\frac{1}{1+x^2+x}\right) \qquad = \tan^{-1}\left(\frac{1}{x^2+x+1}\right)$$

(ii) obtain the function whose first difference is $2x^3 + 3x^2 - 5x + 4$

Solution $\Delta f(x) = 2x^3 + 3x^2 - 5x + 4$ **[2015 Q. 6(a)]**

$= A[x]^3 + B[x]^2 + C[x] + D$

We know

	x^3	x^2	x	
1	2	3	-5	4=D
	0	2	5	
	2	5	0=c	
2	0	4		
	2=A	9=B		

$\Delta f(x) = 2[x]^3 + 9[x]^2 + 0.[x] + 4$

$\Rightarrow \Delta f(x) = 2[x]^3 + 9[x]^2 + 4$

Integrating both sides

$$\Rightarrow f(x) = 2\frac{[x]^4}{4} + 9\frac{[x]^3}{3} + 4[x] + k$$

$$= \frac{1}{2}[x]^4 + 3[x]^3 + 4[x] + k$$